

# DESIGN AND SIMULATION OF VARIANTS OF KOCH MODELS FOR FRACTAL ANTENNA

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# SUPERIOR FRACTAL ANTENNAS

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## ABSTRACT

Fractal antennas are multi-resonant, wideband and smaller in size and have good applications in areas like mission-oriented activities. Many fractal shapes work better as antenna than circular antenna. A good antenna is mainly measured in terms of return loss (RL), Voltage Standing Wave Ratio (VSWR) and bandwidth. The performance of a fractal antenna is directly related to its dimension. Koch curve and many of its variants have been studied widely as fractal antennas and their antenna characteristics have been improved. There is a continuous need to improve the performance of fractal antenna in terms of reduced size, lesser VSWR, lesser RL and wider bandwidth. For improved performance, shapes of further lesser dimensions with smaller sizes are required. It is also required to have more flexibility in design of fractal antennas of smaller sizes. The self-similarity dimension of the Koch curve geometry can be varied by changing the generator used in the recursive Iterated Function System (IFS). In the research work, new variants of Koch models have been generated by dividing the initiator into unequal parts and their mathematical properties have been analyzed. Many of the new variants of Koch models are of reduced size. One of the reduced-sized Koch model have been simulated and it was found that it has better performance than the von Koch curve in terms of RL, VSWR and bandwidth. One of the new variants, which is of the same size and dimension as conventional Koch curve but with different design, have been generated, simulated and compared with the conventional Koch curve. It was found that the two curves of the same size and dimension performed differently due to different designs.

## ABSTRAK

Antena fraktal mempunyai pelbagai salunan, berjalur lebar dan bersaiz lebih kecil serta mempunyai aplikasi yang baik dalam bidang yang berorientasikan misi. Kebanyakan antena yang berbentuk fractal berfungsi lebih baik daripada antena berbentuk bulat. Antenna yang baik diukur dari segi *return loss (RL)*, *Voltage Standing Wave Ratio (VSWR)* dan jalurlebar. Prestasi antena fraktal berkait secara langsung dengan dimensinya. Pelbagai jenis lengkung *Koch* telah dikaji secara meluas sebagai antena fractal dan ciri-ciri antenna telah bertambah baik. Terdapat keperluan yang berterusan untuk meningkatkan prestasi antena fraktal dari segi saiz yang lebih kecil, *VSWR* yang kurang, *RL* yang lebih rendah dan jalurlebar yang lebih luas. Untuk prestasi yang lebih baik, bentuk dimensi yang kurang serta saiz yang lebih kecil diperlukan. Reka bentuk antena fraktal yang bersaiz lebih kecil juga memerlukan lebih fleksibiliti. Dimensi diri keserupaan geometri lengkung *Koch* boleh diubah dengan menukar penjana yang digunakan dalam fungsi sistem berintegrasi yang rekursif. Dalam kerja penyelidikan yang dijalankan, varian baru model *Koch* telah dihasilkan dengan membahagikan pemula ke beberapa bahagian yang tidak sama dan sifat matematikanya telah dianalisa. Kebanyakan varian baru model *Koch* bersaiz lebih kecil. Salah satu daripada model *Koch* yang bersaiz lebih kecil telah disimulasi dan ianya didapati lebih baik daripada lengkung *von Koch* dimana *RL* berkurangan, *VSWR* lebih rendah dan jalurlebar yang lebih luas. Salah satu varian baru, yang bersaiz dan dimensi yang sama dengan lengkung *Koch* konvensional tetapi dengan reka bentuk yang berlainan telah dihasilkan, disimulasi dan dibandingkan dengan lengkung *Koch* konvensional. Didapati bahawa kedua-dua lengkung yang bersaiz dan berdimensi sama, mempunyai keupayaan yang berbeza kerana reka bentuk yang berlainan.

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**LIST OF ABBREVIATIONS**

CST	Computer Simulation Technology
dB	Decibel
HPBW	Half Power Bandwidth
IFS	Iterated Function System
SO	Superior Iterates
VSWR	Voltage Standing Wave Ratio

## **CHAPTER 1**

### **INTRODUCTION**

#### **1.1 INTRODUCTION**

Modern telecommunication systems require antennas with wider bandwidths and smaller sizes than conventional antennas. This had initiated antenna research in various directions, one of which is by using fractal shaped antenna elements. In recent years of development in antenna area for communication, fractal geometry has occupied front of interest. All the basic trigonometric shapes are already utilized in antenna design and their radiation mechanisms have been well explored in communication. Use of fractals as antennas offers better radiation pattern in communication and offers more controlling parameters to designer.

##### **1.1.1 Fractal Geometries in Antenna**

Several antenna configurations based on fractal geometries have been reported in recent years. Antennas with reduced size have been obtained using Hilbert curve fractal geometry. Furthermore, more design equations for these antennas are obtained in terms of its geometrical parameters such as fractal dimension. Antenna properties have also been linked to fractal dimension of the geometry.

Fractal antennas are multi-resonant and smaller in size. Qualitatively, multi-band characteristics have been associated with the self-similarity of the geometry and Hausdorff dimensions are associated with size. Extensive researches are going on towards quantitative relation between antenna properties and fractal parameters in

communication. Any variation in fractal parameters has direct impact on the primary resonant frequency of the antenna, input resistance at primary resonant frequency, and the ratio of the first two resonant frequencies. In other words, these antenna features can be quantitatively linked to the fractal dimension of the geometry. Fractal antennas with less self-similarity dimension give better radiation pattern. This finding can lead to increased flexibility in designing antennas for communication using fractal geometries. These results have been experimentally validated by the researchers (Vinoy et al., 2003, 2004).

### 1.1.2 The von Koch Curve

Initiator of the von Koch curve is a straight line. Fractal dimension of the von Koch curve is  $\approx 1.262$ . The Koch curve is a graphical curve, which is continuous everywhere but not differentiable at any point. The length of Koch curve is infinite. When Koch curve is generated on initiator triangle, Koch snowflake curve will be obtained. The beauty of the Koch snowflake is that its perimeter is infinite but area is compact and equal to  $\frac{\sqrt{3}}{4}r^2$ , where  $r$  denotes the radius of the circle which accommodates the Koch snowflake. This geometrical property makes the Koch fractal as better antenna in place of circular antenna (Falconer, 2003).

### 1.1.3 Superior Fractals

In 2002 (Rani), two-step feedback machine have been implemented in fractal graphics via superior iterations and generated superior fractals. Since then a few fractal models have been improved based on the idea of unequal division of an initiator and thus the gallery of superior fractals have been enriched. New fractal carpets, fractal plants and superior Cantor sets are examples of superior fractals generated by unequal division of initiator.

Above mentioned superior fractal models suggest that there exists a whole gallery of fractal models for the same kind of problem rather than one model, and they have different fractal dimensions. For example, there used to exist a Cantor set and recently many superior Cantor sets have been suggested. In this new gallery of Cantor

sets, most of the objects have different fractal dimensions. Kumar (2010) gave a detailed study on superior fractals in his Master thesis and in future work, he suggested generation of superior Koch curves and their use as antenna.

## **1.2 MOTIVATION**

The von Koch curve has been rigorously analyzed and widely used as fractal antenna. The performance of a fractal antenna is directly related to its dimension. Antennas of lesser dimension give better performance. To improve the antenna characteristics of the Koch curve, a few variants of Koch curve of lesser dimension has been given. Barcellos (1984) gave variants of Koch curve by dividing the initiator into 4 equal parts. These curves have a fixed dimension. Further, there was no suggestion to obtain the curves of lesser dimension. Vinoy, Jose and Vardan (2002) generated new shapes of Koch antenna by varying its indentation angle. All these variants have lesser dimension than the conventional Koch antenna. The curves given by Vinoy et al. occupy more area than the conventional Koch curve. So there is a need to propose Koch shapes of lesser dimension and compact size for better performance with compact in size. Also, there is a curiosity left in the literature whether the two different shapes with the same dimension will have the same performance as antenna or not.

## **1.3 PROBLEM STATEMENT**

Fractal antennas are smaller in size, multi-resonant and wideband. Koch curve and many of its variants have been rigorously studied as fractal antennas and gradually their antenna characteristics have been improved. Few variants have a fixed lesser dimension and few variants have variable lesser dimensions but their area is wider than the conventional Koch curve. There is a continuous need to improve the performance of fractal antenna. For improved performance, shapes of further lesser dimensions with smaller sizes are required. In addition, it is required to have more flexibility in design of fractal antennas of smaller sizes.



## **1.4 OBJECTIVES OF THE STUDY**

The objectives of the study are

- i. To propose new design of Koch models by unequal division of the initiators, and measure their geometrical properties to be used as fractal antennas.
- ii. To simulate the performance of the new design shapes as antenna.

## **1.5 OUTCOMES OF THE STUDY**

The outcomes of the study are as follows:

- i. Design of new variants of Koch curves and Koch snowflakes, and their classification.
- ii. Rewriting systems for generation of new variants of Koch curves.
- iii. Formulas to measure the geometrical properties of new variants of Koch fractals.
- iv. A better Koch fractal antenna will be obtained.

## **1.6 ORGANIZATION OF THESIS**

The thesis has been organized as follows:

Chapter 1 is introductory in nature. It explains the problem statement, objective of the work and related details of Koch curve.

Chapter 2 provides the basic concepts and literature review of fractals and antennas, which forms the basis of the research work.

In chapter 3, new antenna geometries that are new examples of superior fractals have been proposed and their productions rules have been defined. Also, new formulas have been given to measure the geometrical properties of new fractals.

In chapter 4, two new fractal antenna designs have been simulated and compared with the conventional antennas.

Finally, the research work has been concluded in Chapter 5. Also, future work has been suggested.

## **CHAPTER 2**

### **LITERATURE REVIEW**

#### **2.1 INTRODUCTION TO FRACTALS**

Large number of people believes that the geometry of nature is centered on simple figures such as a lines, circles, conic sections, polygons, sphere, quadratic surfaces and so on. For example, tyres of the vehicle are circular, solar system moves around the sun in elliptical orbit. Poles are cylindrical, etc. Have we ever thought, what is the shape of a mountain? Can we describe the structure of animals and plants? How can the networks of veins that supply blood be described by classical geometry? Many objects in nature are so complicated and irregular that it is hopeless to use classical geometry to model them. To analyze many of these questions fractals and mathematical chaos are the appropriate tools.

##### **2.1.1 Definition**

Benoît Mandelbrot coined the term “fractal” in 1975. The term “fractal” is derived from the Latin word “fractus”, which means "broken" or "fractured". A fractal is defined as a rough or fragmented geometric shape that can be divided into smaller parts and each of the parts is a reduced-size copy (or at least approximately) of the whole (Mandelbrot, 1982; Crownover, 1995). A mathematical fractal is based on an equation that undergoes iteration, a form of feedback based on recursion (Briggs, 1992).

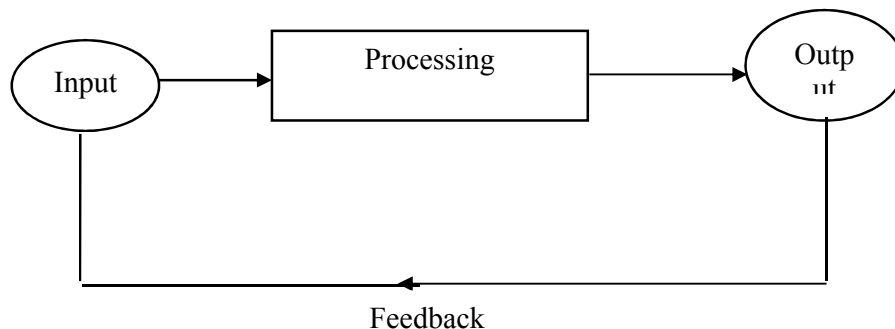
Since, fractals are scale invariant, i.e., fractals appear similar at all levels of magnification, fractals are often considered to be infinitely complex objects (in informal terms). Examples of natural objects that are approximate fractals are: various vegetables (cauliflower and broccoli), coastlines, animal coloration patterns, clouds, snowflakes, mountain ranges, lightning bolts. However, not all self-similar objects are fractals. For example, the real line (a straight line) is formally self-similar but fails to have other fractal characteristics. It is irregular enough to be described in Euclidean terms (Crilly, 1991; Edgar, 1990).

A fractal often has the following features:

- i. A fractal can not be described by traditional Euclidean geometric language as it is too irregular.
- ii. It is self-similar (at least approximately).
- iii. It is scale invariant, i.e., It has a fine structure at arbitrarily small scales.
- iv. It has dimension in fractions.
- v. It has a simple and recursive definition.

### 2.1.2 Backbone of the Fractals

Whole fractal theory is based on the feedback process. Feedback processes are fundamental in all sciences and has become backbone of fractal theory. Sir Issac Newton and Gotfried W. Leibnitz, first, introduced feedback processes independently about 300 years ago in the form of dynamical law. It is simple in principle. The same process is performed repeatedly with new input (Petigen et al., 2004). The output of a function at some iteration is the input for the next iteration. It can be easily understood by the following Figure 2.1.

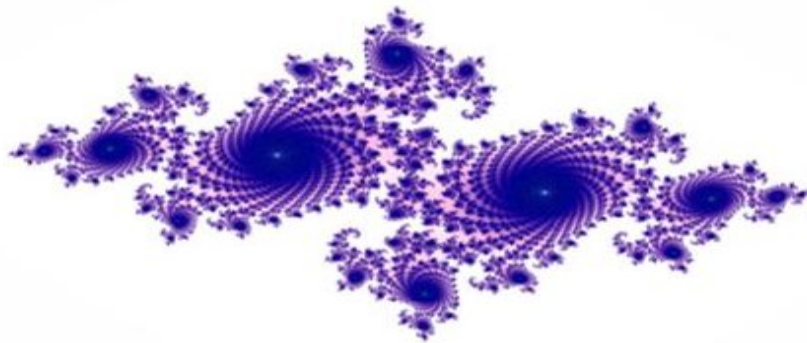


**Figure 2.1:** Feedback Process (Petigen et. al., 2004)

### 2.1.3 Generation

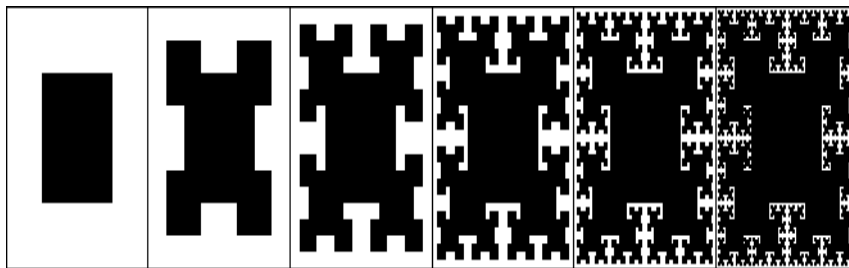
There are four common techniques for generation of fractals:

- i. Escape-time fractals: These are also known as "orbits" fractals, and defined by a complex functions. Mandelbrot sets, Julia sets, Burning Ship fractal, Nova fractal, the Lyapunov fractal, Biomorphs etc. are some popular examples of escape-time fractals. A Julia set, an example of escape-time fractal is shown in Figure 2.2 (Peitgen et. al., 2004).



**Figure 2.2:** A Julia Set: An Example of Escape Time Fractal (Peitgen et. al., 2004)

- ii. Iterated function systems (IFS): These fractals have a fixed geometric replacement rule. Examples of this type are the Cantor sets, fractal carpets, Sierpinski gaskets, Peano curve, Koch snowflakes, Harter-Highway dragon curve, T-square and Menger sponge. An example of IFS fractal is shown in Figure 2.3 (Crownover, 1995).



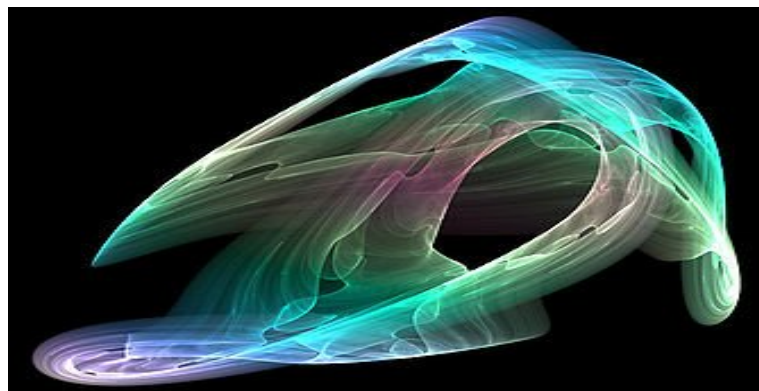
**Figure 2.3:** T-Square Fractal: An Example of IFS Fractal (Crownover, 1995)

- iii. Random fractals: Random fractals are generated by stochastic rather than deterministic processes. Examples of these types are trajectories of the Brownian motion, Lacvy flight, fractal landscapes and the Brownian tree. A fractal landscape is shown in Figure 2.4 (Crownover, 1995).



**Figure 2.4:** Fractal Landscape: An Example of Random Fractal (Crownover, 1995)

- iv. Strange attractors: These kind of fractals are generated by iterating a map or the solution of a system obtained by iterating initial-value differential equations that exhibit chaos. See a strange attractor in Figure 2.5 (Peitgen et al., 2004).



**Figure 2.5:** Visual Representation of a Strange Attractor (Peitgen et al., 2004)

### 2.1.4 Classification

Fractals can be classified according to their self-similarity. There are three types of self-similarity found in fractals (Peitgen et al., 2004; Crownover, 1995 and Schroeder, 1991):

- i. Exact self-similarity
- ii. Quasi-self-similarity
- iii. Statistical self-similarity

**Table 2.1:** Self-Similarity Classification

	<b>Exact self-similarity</b>	<b>Quasi-self-similarity</b>	<b>Statistical self-similarity</b>
<b>Form of self-similarity</b>	The strongest type of self-similarity	Loose form of self-similarity	The weakest type of self-similarity
<b>Self-similarity at scales</b>	Fractal appears identical at different scales.	Fractal appears approximately (but not exactly) identical at different scales. Quasi-self-similar fractals contain small copies of the entire fractal in distorted and degenerate forms.	Fractal has numerical or statistical measures which are preserved across scales.
<b>Example</b>	Fractals defined by iterated function systems often display exact self similarity.	Fractals defined by recurrence relations are usually quasi-self-similar but not exactly self-similar.	Random fractals are examples of fractals which are statistically self-similar, but neither exactly nor quasi-self-similar.

### 2.1.5 Applications

The best thing about fractals is the variety of their applications, e.g., fractal antenna (Cohen, 1996), fractals in nature (Mandelbrot, 1982; Barnsley, 2006), fractals in olden days as Meru or Pascal triangle (Patwardhan, Naimpally & Singh, 2001; Rani, 2005), fractal architecture (Jackson, 2004), fractal weapon (Rani & Kumar, 2002), fractal noise (Rani & Agarwal, 2010), fractals image coding (Wohlberg & Jager, 1999) etc. The applications of fractals are found in every small or large parts of the universe, i.e., from bacteria cultures to galaxies to our body. Barnsley and Hawley (1993) wrote a book “Fractals Everywhere” in which they suggested applications of fractals in a lot of areas.

A list of application areas of fractals apart from the above mentioned areas are diffusion, economy, fractal art, fractal music, landscapes, Newton's method, special effects, weather, galaxies, rings of Saturn, bacteria cultures, chemical reactions, human anatomy, molecules, Botany, population growth, clouds, coastlines and borderlines, Chemistry, medical science, film industry, wavelet theory, nanotechnology etc (see for instance, Bunde, Armin & Havlin, 1994, Peitgen, Henriques, Penedo, 1991; Peitgen & Richter, 1986 and Petigen & Saupe, 1988).

## 2.2 FRACTAL DIMENSION

At the turn of the nineteenth century, it was one of the major problems in science to determine what dimension means and which properties it has. Mathematicians have come up with ten different notions of dimension: self-similarity dimension, box-counting dimension, Hausdorff dimension, topological dimension, fractal dimension, information dimension, capacity dimension and more. Some of them make sense in certain situations but not in other situations. However, in certain situations, they all make sense and all are the same. Some are used very often and easily in fractal theory. However, all the different dimensions are special forms of Mandelbrot's fractal dimension (Peitgen, Jurgen & Saupe, 2004, p. 202). Here, self-similarity dimension is explained in brief, which is one of the most popular dimension methods in fractal graphics.

### Self-Similarity Dimension

This is generally used in calculating dimension of self-similar figures. There is a nice power relation between the number of pieces  $n$  of an object and the reduction factor  $s$  as shown in Eq. (2.1) and Eq. (2.2).

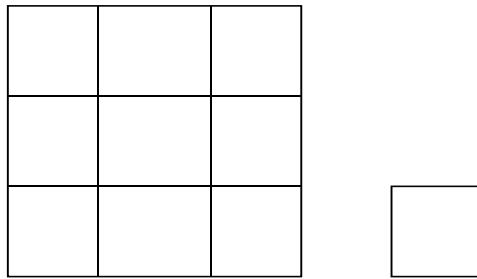
$$n = 1/s^D \quad (2.1)$$

where  $n$  is the number of pieces of an object,  
 $D$  is the dimension of the object and  
 $s$  is the scaling factor.

From Eq. (2.1), following equation is obtained:

$$\log n = \log(1/s^D) \quad (2.2)$$

For example, in following square, apply  $s = 1/3$  then  $n = 9$



So  $\log 9 = D \log 3$  and  $D = 2$ . Thus the dimension of square is 2.

### 2.3 FRACTAL ANTENNAS

In today's world of wireless communications, there has been an increasing need for more compact and portable communications systems. Just as the size of circuitry has evolved to transceivers on a single chip, there is also a need to evolve antenna